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ROYAL AIR FORCE—AIRCRAFT APPRENTICES

No. 1 RADIO SCHOOL, LOCKING

FINAL EXAMINATION IN EDUCATIONAL SUBJECTS

MAY, 1955 (80TH) ENTRY

MARCH, 1957

MATHEMATICS I AND MECHANICS

Time allowed—Three hours

Candidates are to answer FOUR questions from Section A and TWO questions from Section B

(ALL questions carry equal marks)

SECTION A—MATHEMATICS I

I. (a) Factorize :—

(i) $x^6 - 64$

(ii) $4x^4 - 13x^2 + 9$

(iii) $2xy - 21pq - 14xp + 3qy$

(b) Solve the equations :—

(i) $\frac{1}{x - \frac{1}{x}} = \frac{12}{7}$

(ii) $x(x - 3) = x - 1$

(c) Express the complex number $1 + \sqrt{3}j$ in the $r \angle \theta$ form.

OVER

2. (a) Expand by the Binomial Theorem as far as the term in x^4 the following :—

(i) $(1 + x)^{12}$

(ii) $(1 + x)^{-2}$

(b) (i) Show that if $x = \frac{1}{50}$ then

$$\frac{1}{\sqrt{1-x}} = \frac{5\sqrt{2}}{7}.$$

(ii) Using the expansion of $(1 - x)^{-\frac{1}{2}}$ as far as the term in x^3 , and putting $x = \frac{1}{50}$ find a value for $\sqrt{2}$.

(c) The last term of an Arithmetic Progression is 10 times the first, and the last but one is equal to the sum of the 4th and 5th terms. Show that the first term is equal to the common difference of the series.

3. (a) Simplify :—

(i) $\frac{a^4 (b^2 c^3)^3}{(a^2 bc)^6}$.

(ii) $(j - 1)^4$

(iii) $\frac{(6x^2 + x - 1)(2x^2 - 9x + 4)}{3x^2 - 13x + 4}$.

(iv) $\frac{2^{n+3}}{(2^n)^{n-1}} \div \frac{4^{n+1}}{(2^{n-1})^{n+1}}$.

(b) Without using tables find the value of :—

(i) $\log_{10} 360 + \log_{10} 5 - \log_{10} 18$

(ii) $\log_9 27$

(iii) $\log_3 9$

4. (a) Represent the following vectors on an Argand diagram :

(i) $3 - 4j$ (ii) $5 \angle 130^\circ$ (iii) -5

(iv) $j(4 - 3j)$ (v) $5 \cos(-100^\circ) + 5j \sin(-100^\circ)$

(b) Simplify $\frac{3(\cos 135^\circ + j \sin 135^\circ)^{\frac{1}{2}}}{\sqrt{36 \cos 60^\circ + 36j \sin 60^\circ}}$.

(c) Given $\sqrt{3} = 1.7320508$, find the value of $\frac{1}{2 - \sqrt{3}}$.

(d) Evaluate $\log_4 5$.

5. (a) If θ is an acute angle and $\tan \theta = \frac{5}{12}$ express $\sin \theta$ and $\sec \theta$ as fractions.
- (b) Write down the expansion of $\cos(A + B)$ and use this to deduce *three* expansions of $\cos 2A$.
- (c) Sketch roughly, for values of x between 0 and 2π , the graphs of :—
- (i) $y = 3 \sin 2x$
 (ii) $y = \frac{1}{2} \cos x$
 (iii) $y = \sin(x + 30^\circ)$
- (d) Prove the identity :—

$$\frac{\sin \theta}{1 + \cot \theta} - \frac{\cos \theta}{1 + \tan \theta} = \sin \theta - \cos \theta.$$

6. (a) The driving wheels of an engine are 6 ft in diameter. How many rev/min do they make when the engine is travelling at 15 m.p.h.

(Take $\pi = \frac{22}{7}$)

(b) Simplify $3\sqrt{125} - 2\sqrt{80} + \sqrt{605}$

- (c) Using the expansion of $\sin(A+B)$ and $\cos(A+B)$ show that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

- (d) Three points A, B and C on level ground are situated so that the distances AB, AC are 1,000 yards and 2,000 yards respectively; the angle BAC being 60° . Calculate, in feet, the altitude of an aircraft which is vertically above C if its angle of elevation from B is 45° .

SECTION B—MECHANICS

7. (a) Define :—

(i) *Impulse.*

(ii) *Momentum*

- (b) Give the equation connecting *work done* and *change in Kinetic Energy*.

- (c) A bullet weighing $\frac{1}{2}$ oz has its velocity reduced from 1,200 ft/s² to 400 ft/s² in passing through timber 6 inches thick. What would be the least thickness of such timber required to stop the bullet? (Assume that the timber exerts a constant retarding force.)

8. (a) A block of mass 10 lb is at rest on a *smooth* horizontal table. When a constant horizontal force is applied to the block it is found to move 18 ft in 3 seconds. Find the applied force in lb wt.

(Take $g = 32 \text{ ft/s}^2$.)

- (b) An aircraft describes a horizontal circle of radius 1,000 feet with a speed of 120ft/s. At what angle is it banked?
- (c) A model aircraft is constrained to fly in a horizontal circle by means of a wire of length 8 yd. The weight of the aircraft is $\frac{3}{4}$ lb and it makes 40 complete revolutions per minute. Find the tension in the wire. (Take $\pi^2 = 10$.)
9. (a) State *Newton's Second Law of Motion*.
- (b) A man weighing 150 lb is standing in a helicopter. Find the thrust exerted by the man on the floor of the helicopter when :—
- the helicopter rises vertically with an acceleration of 4 ft/s^2 ,
 - the helicopter descends vertically with an acceleration of 4 ft/s^2 .